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OPTICAL PROPERTIES OF AEROSOL PARTICLES: A REVIEW OF APPROXIMATE ANALYTICAL SOLUTIONS

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Abstract—Approximate solutions for integral light scattering characteristics (efficiency factors, asymmetry parameter) of spherical aerosol particles and their polydispersions were reviewed. All formulas are accompanied with an accuracy estimation and a range of applicability. Copyright © 1996 Elsevier Science Ltd

NOTATION

- a radius of a particle
- a_{ef} effective radius
- C_{abs} absorption cross section
- C_{ext} extinction cross section
- C_{pr} light pressure cross section
- C_{sca} scattering cross section
- C_v volumetric concentration
- f particle size distribution
- g asymmetry parameter
- k wave number
- m complex refractive index of particles
- n real part of complex refractive index
- Q_{abs} absorption efficiency factor
- Q_{ext} extinction efficiency factor
- Q_{sca} scattering efficiency factor
- V volume of a particle
- x size parameter
- x_{ef} effective size parameter

Greek letters

- θ scattering angle
- κ imaginary part of the complex refractive index
- λ wavelength of light
- ρ phase shift
- Σ geometrical cross section of a particle
- σ_{abs} absorption coefficient
- σ_{ext} extinction coefficient
- ω albedo of a particle

1. INTRODUCTION

Different optical techniques are used for aerosol characterization and particle sizing (Junge, 1963; Ivanov, 1969; Wikramasinghe, 1973; McCartney, 1977; Kondratyev and Binenko, 1984; Shifrin, 1988; Stanley-Wood and Lines, 1992; Shifrin and Tonna, 1993).

Very often the optical methods are the only way to study aerosol properties (astronomic observation, solar photometry, satellite remote sensing, etc.).

Approaches relating the optical properties of aerosol to its microstructure parameters and the complex refractive index form the theoretical basis of aerosol optics. These approaches are developed in the theory of light scattering by small particles. A lot of fundamental monographs (e.g. Shifrin, 1951; van de Hulst, 1957; Kerker, 1969; Bohren and Huffman, 1983) and reviews (Logan, 1965; Saxon, 1955) deal with this problem.

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We will consider here only spherical scatterers. Such particles are a very common occurrence in natural media (clouds, fogs, some aerosols and hydrosols, etc.). Moreover, in many cases the optical properties of slightly nonspherical particles (e.g. such as dust and salt scatterers) can be modeled by spherical ones (Schuerman, 1983).

The problem of light diffraction by spheres was solved as long back as in the last century (Love, 1899; Mie, 1908; Debye, 1909) and just now the Mie computation codes (Deirmendjian, 1969; Prishivalko and Naumenko, 1972; Lentz, 1976; Wiscombe, 1979, 1980; Bohren and Huffman, 1983; Prishivalko *et al.*, 1984; Wang and van de Hulst, 1991) are available for all scientists. But in some aerosol optics problems (e.g. inverse problems) the preference should be given to the straightforward approximate solutions. It is not occasionally, that different approximations are given in all classical monographs on single light scattering and numerous new approaches have been developing currently. Unfortunately, these results are spread over journals of different type. A lot of them have never been included in the monographs and reviews. It is about a time to bring together different approximate solutions for local optical properties of aerosol media, to consider the old and new results from one point of view and to develop some reference stuff for researchers and engineers who deal with various aerosol optics problems. To do so is a main goal of this paper.

As known, the main scattering characteristics of a particle are the albedo ω, the extinction cross section C_{ext}, the absorption cross section C_{abs} and the phase function $p(\theta)$, θ is the scattering angle. All of them are determined by the size parameter x and the complex refractive index m:

$$x = ka, \quad m = n - \mathrm{i}\kappa, \tag{1}$$

where a is the radius of a particle, $k = 2\pi/\lambda$, λ is the wavelength.

Different approximations of the light scattering problem arrive at small and large values of x and $(m-1)$.

1. At $x \to 0$, $|mx| \to 0$, the internal field in a particle is uniform, which provides the Rayleigh approximation. The absorption cross section C_{abs} is proportional to the particle's volume, scattering cross section C_{sca} is proportional to the squared volume of the particle. The phase function is given by the well known simple function $p(\theta) = 0.75\ (1 + \cos^2\theta)$.
2. At $x \to \infty$, $2x|m-1| \to \infty$, the geometrical optics approximation can be applied. In this case, the extinction cross section $C_{\text{ext}} = 2\Sigma$, where Σ is the geometrical cross section of a particle. The absorption cross section C_{abs} is determined by the attenuation parameter $c = 4\kappa x$. For strongly absorbing particles ($c \gg 1$) we have $C_{\text{abs}} \sim \Sigma$ while for weak absorption ($c \ll 1$) $C_{\text{abs}} \sim V$, V is the particle's volume. The phase function is extremely forward extended, and the average cosine of the scattering angle is slightly different from 1.
3. At $|m-1| \to 0$, there are three distinct approximations: Rayleigh–Gans (any x, $\rho = 2x|m-1| \to 1$), van de Hulst ($x \gg 1$, any ρ), and Perelman (any x and ρ).
4. At moderate x and n values $x > 10$, $n > 1.1$), the complex angular momentum theory can be used (Nussenzveig, 1969a, b). This theory gives wave corrections to the geometrical optical solutions.

All these approximations provide analytical solutions for optical characteristics, which will be considered in this paper. We will make emphasis on the formulas for large particles, because coarse aerosols are a common occurrence. Moreover, the larger the particles the more cumbersome Mie computations are.

Thus, we will bring together here the straightforward solutions for the efficiency factors $Q_{\text{ext}} = C_{\text{ext}}/\Sigma$, $Q_{\text{abs}} = C_{\text{abs}}/\Sigma$ and asymmetry parameter of the phase function g:

$$g = \frac{1}{2}\int_0^\pi p(\theta)\sin\theta\cos\theta\,\mathrm{d}\theta. \tag{2}$$

The phase function is supposed to be normalized by the following condition:

$$\frac{1}{2}\int_0^\pi p(\theta)\sin\theta\,\mathrm{d}\theta = 1. \tag{3}$$

It is impossible to give a full description of the different approximations for phase functions within a frame of a paper. So we have chosen the value of g as the most representative and commonly used parameter of the phase function (see also the beginning of the Section 4).

In terms of these parameters, one can obtain the radiation pressure cross section

$$C_{pr} = (Q_{ext} - Q_{sca}g)\pi a^2, \tag{4}$$

the scattering cross section

$$C_{sca} = (Q_{ext} - Q_{abs})\pi a^2, \tag{5}$$

and the albedo

$$\omega = 1 - Q_{abs}/Q_{ext}. \tag{6}$$

Most aerosols are polydispersed media. The asymmetry parameter $\langle \cos\theta \rangle$, the extinction σ_{ext} and the absorption σ_{abs} coefficients of spherical polydispersions are given by the following equations (van de Hulst, 1957; Marov *et al.*, 1989):

$$\langle \cos\theta \rangle = \frac{\int_0^\infty f(a) g C_{sca}\,\mathrm{d}a}{\int_0^\infty f(a) C_{sca}\,\mathrm{d}a} \tag{7}$$

$$\sigma_{ext} = N\int_0^\infty f(a) C_{ext}\,\mathrm{d}a, \tag{8}$$

$$\sigma_{abs} = N\int_0^\infty f(a) C_{abs}\,\mathrm{d}a, \tag{9}$$

where $f(a)$ is the particle size distribution (PSD).

The usual way to present results of different approaches is to consider them separately and one by one. We prefer here another scheme. To make the use of our paper more comfortable we gather all approximations for the extinction efficiency factor in Table 1 with explanations and accuracy estimations in Section 2. Similarly Table 4 and Section 3 are devoted to the absorption efficiency factor. The formulas for the asymmetry parameter are given in Table 5 and commented in Section 4. In Section 5 integral optical characteristics of water clouds are considered.

2. EXTINCTION EFFICIENCY FACTOR

The basic approximate solutions for the extinction efficiency factor are given in Table 1. The following gives the needed brief explanations.

2.1. *Rayleigh approximation*

The Rayleigh approximation provides the optical properties of particles which are much smaller than the wavelength ($x \ll 1$, $|mx| \ll 1$). In this case, the internal field, originated in a particle by the electromagnetic wave, is uniform inside a particle. So, this problem can be reduced to the standard electrostatic problem which is an isotropic, homogeneous, dielectric sphere in a uniform field (Kerker, 1969). In this approximation with regard to the terms of the order of x^4, it was obtained by Pendorf (1960):

$$Q^{R}_{ext} = \frac{24n\kappa}{F_1(n,\kappa)}x + \left\{\frac{4n\kappa}{15} + \frac{20n\kappa}{3F_2(n,\kappa)} + \frac{4.8n\kappa[7(n^2+\kappa^2)^2 + 4(n^2-\kappa^2-5)]}{F_1^2(n,\kappa)}\right\}x^3$$
$$+\frac{8}{3}\left\{\frac{[(n^2+\kappa^2)^2 + (n^2-\kappa^2-2)]^2 - 36n^2\kappa^2}{F_1^2(n,\kappa)}\right\}x^4, \tag{10}$$

Table 1. The extinction efficiency factor Q_{ext} of a spherical particle ($\tan\beta = \kappa/(n-1)$)

N	Approximation, range of applicability	Q_{ext}
1.	Rayleigh $x \ll 1$, $\lvert mx \rvert \ll 1$	$\frac{8}{3}x^4 \left\lvert \frac{1-m^2}{1+m^2} \right\rvert^2, \quad \kappa = 0$ $4x\,\mathrm{Im}\left[\frac{1-m^2}{2+m^2}\right], \quad \kappa \neq 0$
2.	Geometrical optics, $x \gg 1$, $\rho \gg 1$	2
3.	Geometrical optics with regard to the edge effects, $x > 10$	$2(1 + x^{-2/3})$
4.	Perelman, any x	$\frac{1}{2\lvert m \rvert}\mathrm{Re}\left\{(1+m^2)^2 + \frac{\omega(m,x) - \omega(-m,x)}{2m}\right\}$ for $\omega(m, x)$ see equation (16)
5.	Reyleigh–Gans, $\lvert m-1 \rvert \ll 1$, $x \ll \frac{1}{2\lvert m-1 \rvert}$	$\lvert m-1 \rvert^2 \varphi(x)$, for $\varphi(x)$ see equation (19)
6.	van de Hulst, $x \gg 1$, any $\rho = 2x(n-1)$	$Q_{ext}^{HA} = 2 - 4\rho^{-1}e^{-\rho tg\beta}\cos\beta\sin(\rho-\beta)$ $- 4\rho^{-2}e^{-\rho tg\beta}\cos^2\beta\cos(\rho - 2\beta)$ $+ 4\rho^{-2}\cos 2\beta\cos^2\beta, \quad \kappa \neq 0$ $2 - \frac{4\sin\rho}{\rho} + \frac{4(1-\cos\rho)}{\rho^2}, \quad \kappa = 0$
7.	Evans–Fournier, any x	$Q_{ext}^{R}\left[1 + \left[\frac{Q_{ext}^{R}}{TQ_{ext}^{HA}}\right]^P\right]^{-1/P}$ Q_{ext}^{R} is given by equation (10)

where

$$F_1(n,\kappa) = (n^2 + \kappa^2)^2 + 4(n^2 - \kappa^2) + 4;$$
$$F_2(n,\kappa) = 4(n^2 + \kappa^2)^2 + 12(n^2 - \kappa^2) + 9. \tag{11}$$

The simple formulas given in the first two lines of Table 1 follow directly from equation (10) at $\kappa = 0$ and $\kappa \neq 0$ ($x \to 0$), respectively.

The accuracy of equation (10) was discussed in detail by Heller (1965), Farone and Robinson (1968), and Kerker (1969). The errors of equation (10) do not exceed 15–30% when $x < 1.4$ and $m = 2$ for transparent particles and at $x < 0.8$ (n ranged from 1.25 to 1.75) for absorbing spheres ($\kappa \leqslant 1$). While n increases from 1 to 1.5 the error of the formula in the first line of Table 1 decreases and reaches its minimum at $n = 1.6$–1.8 and essentially increases again with a further growth of the value of n. As an example, the error of the value of Q_{ext} estimation is less than 5% for $a < 0.1\lambda$ and $m = 1.33$ or $a < 0.06\lambda$ and $m = 1.1$ (Heller, 1965). Thus, the Rayleigh approximation is the less accurate the smaller refractive indices (at $n < 1.8$) of aerosol particles are.

2.2. *Geometrical optics approximation and edge effects*

The geometrical optics (GO) approximation ($x \gg 1$, $\rho = 2x\lvert m-1 \rvert \gg 1$) gives the well-known simple formula (Shifrin, 1951; van de Hulst, 1957) for the value of the extinction efficiency factor:

$$Q_{ext} = 2. \tag{12}$$

In this approximation, the light flux incident upon the spherical particle is split into parts which are reflected, refracted and diffracted.

When the value of x is not large enough, the geometrical optics approximation cannot provide sufficiently good accuracy, and the edge effects should be taken into account. It was done in the paper of Nussenzveig and Wiscombe (1980). The authors, using asymptotic representation of the Mie series, obtained the following solution for the extinction efficiency factor

$$Q_{\text{ext}} = 2(1 + \alpha x^{-2/3}) + N(n, \kappa, x), \tag{13}$$

where $\alpha \approx 1$ and $N(n, \kappa, x)$ is the function which decreases much more rapidly than $x^{-2/3}$ as $x \to \infty$. It has got a complicated form.

Following our decision to consider only simple instructive solutions we do not give it here. Note that equation (12) follows from (13) as $x \to \infty$.

The errors of equation (13) do not exceed 1% at $x \geqslant 15$ and 0.1% for $x \geqslant 70$ ($n > 1.1$) for values of the Q_{ext}, averaged over the high-frequency "ripple". Neglecting the function $N(n, \kappa, x)$ and the difference of α from 1, one can obtain from equation (13):

$$Q_{\text{ext}} = 2(1 + x^{-2/3}). \tag{14}$$

The error of Q_{ext} estimation with equation (14) does not exceed 10% for $x > 45$ and $n \geqslant 1.2$ and decreases very rapidly with increasing of the value of x, oscillating near zero. The larger n, equation (14) more accurate. This simple formula appears pretty good for a lot of applied problems (Kokhanovsky and Zege, 1995).

2.3. *Optically soft particles*

Let us consider now soft ($|m - 1| \ll 1$) particles. Soft particles of different sizes are generally responsible for scattering in sea water and biological suspensions. Sometimes atmospheric aerosols can be considered as scattering media with soft particles (e.g. ice crystals and water drops in the near infrared region of spectrum). The Rayleigh–Gans and the van de Hulst approximations are the most habitual for describing the optical properties of the soft particles. But currently the soft (Perelman, 1986) and the eikonal (Klett and Sutherland, 1992) approaches are attracting more and more users.

2.3.1. *Perelman approximation.* Perelman approximation, sometimes referred as the *soft approximation* (SA), is based on the asymptotic properties of Mie series as $|m - 1| \to 0$ (Perelman, 1986, 1991). The following solution for Q_{ext} at any x was given in this approach (Perelman, 1986, 1991):

$$Q_{\text{ext}} = \frac{1}{2|m|} \operatorname{Re}\left\{(m^2 + 1)^2 + \frac{\omega(m, x) - \omega(-m, x)}{2m}\right\}, \tag{15}$$

where

$$\omega(m, x) = (m^2 - 1)^2\left(m^2 + 1 - \frac{1}{2x^2}\right)\int_0^\rho \frac{1 - e^{-it}}{t}\,dt + (m^2 + 1)^2$$
$$\left\{i(m^4 - 2m^3 - 2m^2 - 2m + 1)\frac{e^{i\rho}}{\rho} + (m^4 - 6m^3 - 14m^2 - 6m + 1)\frac{1 - e^{i\rho}}{\rho^2}\right\}, \tag{16}$$

and $\rho = 2x(n - 1)$. It gets simpler at $\kappa = 0$ (see Granovsky and Ston, 1994, as well):

$$Q_{\text{ext}} = \frac{1}{2n}\left\{(n^2 + 1)^2 + \frac{h(n, \rho) - h(-n, R)}{2n}\right\}, \tag{17}$$

where

$$h(n, z) = \left(a(n) + \frac{a_0(n)}{z^2}\right) Ci(z) + a_1(n)\frac{\sin(z)}{z} + a_2(n)\frac{1-\cos(z)}{z^2},$$
$$a(n) = (n^2-1)^2(n^2+1), \qquad a_0(n) = -2(n^2-1)^2(n-1)^2, \tag{18}$$
$$a_1(n) = (n+1)^2(n^4 - 2n^3 - 2n^2 - 2n + 1),$$
$$a_2(n) = -a_0(n) - a_1(n).$$

The value of $Ci(z)$ is the cosine integral

$$Ci(z) = \int_0^z \frac{1-\cos t}{t}\,dt.$$

The Rayleigh formula for the value of Q_{ext} given in the first line of Table 1 can be derived from equation (17) as $x \to 0$. At small ρ equation (17) produces the Rayleigh–Gans solution (van de Hulst, 1957; Kerker, 1969) and for $x \to \infty$ and $|m-1| \to 0$ coincides with the van de Hulst approximation (van de Hulst, 1957). It provides high accuracy at any x when $|m-1| \ll 1$. That is an essential advantage of this approach in comparison with the Rayleigh–Gans and the van de Hulst methods. The accuracy of equation (17) for n ranged from 1.02 to 1.34 and any $x < 1000$ was studied by Perelman (1986). For instance, the relative error of this approximation is less than 5% at $n \leqslant 1.22$ and $x \leqslant x_0(n)$, where the function $x_0(n)$ is given in Table 2.

2.3.2. *Rayleigh–Gans approximation. The Rayleigh–Gans approximation* (the analog of the Born approximation of quantum mechanics) is applicable for any x if the phase shift $\rho = 2x(n-1)$ is small (van de Hulst, 1957). In this approximation, an arbitrarily shaped particle is subdivided into small elements. Each element is treated as a Rayleigh scatterer excited by the incidence field, which is assumed to be unperturbed by the presence of the rest of the particle. In doing so, one can obtain (van de Hulst, 1957; Kerker, 1969; Newton, 1969):

$$Q_{ext} = |m-1|^2 \varphi(x), \tag{19}$$

where

$$\varphi(x) = 2.5 + 2x^2 - \frac{\sin 4x}{4x} - \frac{7(1-\cos 4x)}{16x^2} + \left(\frac{1}{2x^2} - 2\right)(\gamma + \ln(4x) - Ci(4x))$$

and $\gamma = 0.577$ is the Euler's constant. Note that for large particles ($x \to \infty$) $\varphi(x) = 2x^2$ and $Q_{ext} = \rho^2/2$ in this approach. As mentioned above, the more general equation (15) passes into (19) for $\rho \to 0$.

Sometimes (in particular, when the polydispersion characteristics are under consideration) another presentation of $\varphi(x)$ could be more convenient (Shifrin and Ston, 1976):

$$\varphi(x) = \sum_{l=1}^{\infty} (-1)^{l+1} \frac{(4x)^{2l+2}}{(2l+2)} \frac{l^2+l+2}{2(l+2)^2(l+1)}. \tag{20}$$

The following approximation for the function $\varphi(x)$ (20) can be used with the error no more than 3% (Shifrin and Ston, 1976):

$$\varphi(x) = \begin{cases} 1.185x^4(1 - 0.4x^2 + 0.096x^4) & x \in [0, 1], \\ 1.92x^4 - 1.084x, & x \in [1, 2], \\ 2.112x^2 - 1.456x, & x \in [2, 12.5]. \end{cases} \tag{21}$$

Table 2. The function $x_0(n)$ (Perelman, 1986)

n	1.02–1.06	1.08	1.1	1.12	1.14	1.16	1.18	1.22
x_0	∞	800	300	130	90	72	49	32

2.3.3. *Van de Hulst approximation.* Unlike the Rayleigh–Gans approach, the van de Hulst approximation (HA) is applicable for $x \gg 1$ and $|m-1| \ll 1$ at any value of the phase shift ρ (van de Hulst, 1957). In this approximation, the geometrical optics rays are supposed to pass through a particle without any deflection but they can undergo a significant phase shift because of the long path length through a particle. It results in the alteration of the phase of the original wave on the plane beyond the sphere (Borovoi, 1988). The van de Hulst formula for the extinction efficiency factor Q_{ext}^{HA} is given in Table 1. In the geometrical optics limit as $\rho \to \infty$ $Q_{ext}^{HA} \to 2$ and in the Rayleigh–Gans limit as $\rho \to 0$ $Q_{ext}^{HA} \to \rho^2/2$. A maximum of the error of Q_{ext}^{HA} (see Fig. 1) appears at $x \approx \pi/2(m-1)$ (for $x > 1$).

The values of Q_{ext}^{HA} are smaller than those calculated with the Mie theory (see Fig. 1). Nevertheless, the occurrence of extremes on the extinction curve $Q_{ext}(\rho)$ obtained through the HA is rather correct.

The empirical recipe how to improve the accuracy of the HA formula in the region of the first maximum of the curve $Q_{ext}(\rho)$ was given by Klett (1984). He proposed to use the following relation for the extinction efficiency factor:

$$Q_{ext} = BQ_{ext}^{HA}, \tag{22}$$

where

$$B = 1.1 + (n - 1.2)/3. \tag{23}$$

The van de Hulst approximation can be improved to describe small particles as well. To this end one should use the complex angular momentum (CAM) theory (Nussenzveig, 1969a, b, 1976, 1988; Nussenzveig and Wiscombe, 1980, 1991). The accuracy of the HA supplemented with the edge term $2x^{-2/3}$ [see equation (14)] was investigated by Ackerman and Stephens (1987) (see Ackerman and Cox (1988) as well). They found that the relative error of the value of $Q_{ext} = Q_{ext}^{HA} + 2x^{-2/3}$ was less, than 10% at $x_{ef} \geq 12$–19 and n up to 2 (for the gamma PSD). Here x_{ef} is the effective size parameter:

$$x_{ef} = ka_{ef}, \qquad a_{ef} = \frac{\langle a^3 \rangle}{\langle a^2 \rangle}, \tag{24}$$

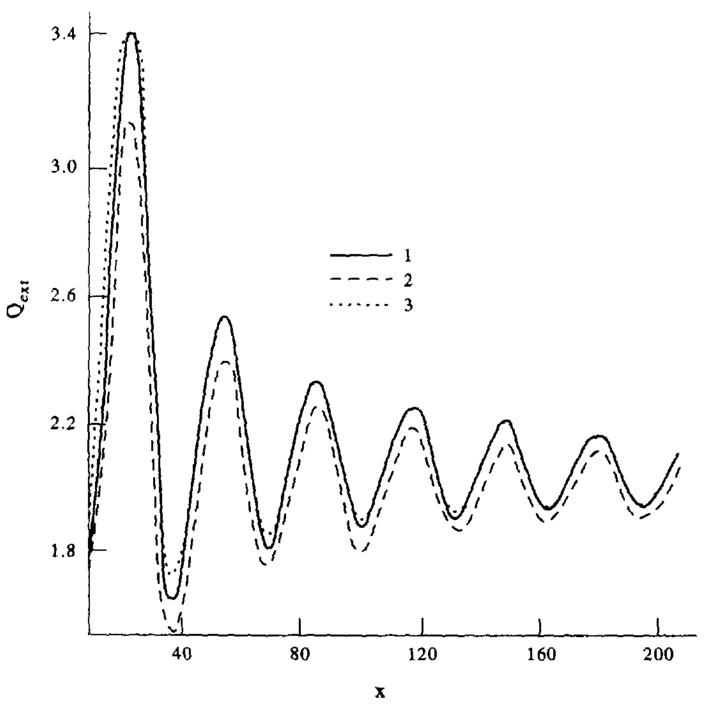

Fig. 1. The extinction efficiency factor Q_{ext} as a function of size parameter x at $n = 1.1$ calculated by the Mie theory (1), the van de Hulst approximation (2), the van de Hulst approximation with regard to the edge effects (3).

where brackets mean the averaging over the PSD $f(a)$:

$$\langle a^n \rangle = \int_0^{\infty} f(a) a^3 \, da. \tag{25}$$

In this approximation the refraction of light by a particle is neglected. The study of Ackerman and Stephens (1987) showed that the regard to the refraction does not practically change the accuracy of the solution.

We carried out special calculations to study the relative error of the HA with the edge term at $m = 1.1, 1.2, 1.34, 153$ and $x = 10(1)210$. It was found, the error is less than 5% for $x > 20$. Meanwhile the error of the HA (without supplementing with the edge term) is larger. For example, it is about 15% at $m = 1.53$ and $x = 20$.

The correction term $x^{-2/3}$ [see equation (14)] is adequate only for $x \geqslant 10$. To be more clear, note that as $x \to 0$ the value of $x^{-2/3} \to \infty$.

It should be pointed out that the main advantage of the HA is the possibility to solve more complex problems such as light extinction by ellipsoids (Greenberg, 1970; Lopatin and Sidko, 1988; Sidko *et al.*, 1990; Paramonov, 1994a, b), layered ellipsoids (Kokhanovsky, 1991b), layered spheres (Chen, 1987; Lopatin and Sidko, 1988; Zege and Kokhanovsky, 1989), anisotropic and optical active particles (Zumer, 1988; Kokhanovsky, 1991a), and crystals (Volkovitski, 1984; Chylek and Klett, 1991; Heffels *et al.*, 1995). It is important that the edge term is mainly determined by a curvature radius of a particle surface (Nussenzveig, 1976). Thus, there is a possibility to use the edge terms obtained for uniform aerosol particles (Nussenzveig and Wiscombe, 1980; Fournier and Evans, 1991) in the case of layered scatterers.

To conclude the section, the eikonal approximation (EA) (Glauber, 1959; Saxon, 1955; Perrin and Lamy, 1984, 1986; Perrin and Chiapetta, 1985) should be mentioned. In this approximation the formula for the value of Q_{ext} is the same as within the framework of the HA (Table 1) but $\rho = (n^2 - 1)ka$ (compare with $\rho = 2ka(n-1)$ within the framework of the HA). Sharma and Somerford (1990) investigated the accuracy of the eikonal approximation (see the paper of Sharma *et al.*, 1988, as well). They found that the errors of the eikonal approximation were less than the errors of the van de Hulst approximation as $\rho < 4$ and $x \leqslant 20$–30. Chen (1989) calculated the accuracy of the eikonal approximation with regard to the edge term (see also Chen, 1984; Chen and Smith, 1992).

2.4. *Evans–Fournier approximation*

We do not consider here numerous empirical formulae for Q_{ext} obtained through approximations of the Mie computation results (Levin, 1961; Deirmedjian, 1969; Sokolik 1989; Adzerikho, 1990; Zhang, 1990; etc.). Only one exception was made. The nice formula derived by Evans and Fournier (1990), which is given in the end of Table 1, sews together the van de Hulst approximation (Q_{ext}^{HA}, see Table 1) for large particles and the Rayleigh approximation (Q_{ext}^{R}, see Table 1) using the empirical functions obtained through the approximation of Mie computation results. This formula is applicable at any x values and at least for $m < 2$. The parameters P and T included (see Table 1) are given by using the following equations:

$$P = A + \mu/x, \qquad T = 2 - \exp(-x^{-2/3}), \tag{26}$$

where

$$A = \frac{1}{2} + \left[n - 1 - \frac{2}{3}\sqrt{\kappa} - \frac{\kappa}{2}\right] + \left[n - 1 + \frac{2}{3}(\sqrt{\kappa} - 5\kappa)\right]^2,$$

$$\mu = \frac{3}{5} - \frac{3}{4}\sqrt{n-1} + 3(n-1)^4 + \frac{25}{6 + (5(n-1)/\kappa)}.$$

At $\kappa = 10^{-6}$–10^{-2} and $n \leqslant 1.62$, the relative error of the Evans–Fournier approximation (EFA) does not exceed 20% at any x even without averaging over the ripple structure of the extinction curve, and essentially decreases for polydispersions. The errors of the value of Q_{ext} within the framework of this approach are plotted in Fig. 2 for typical atmospherical

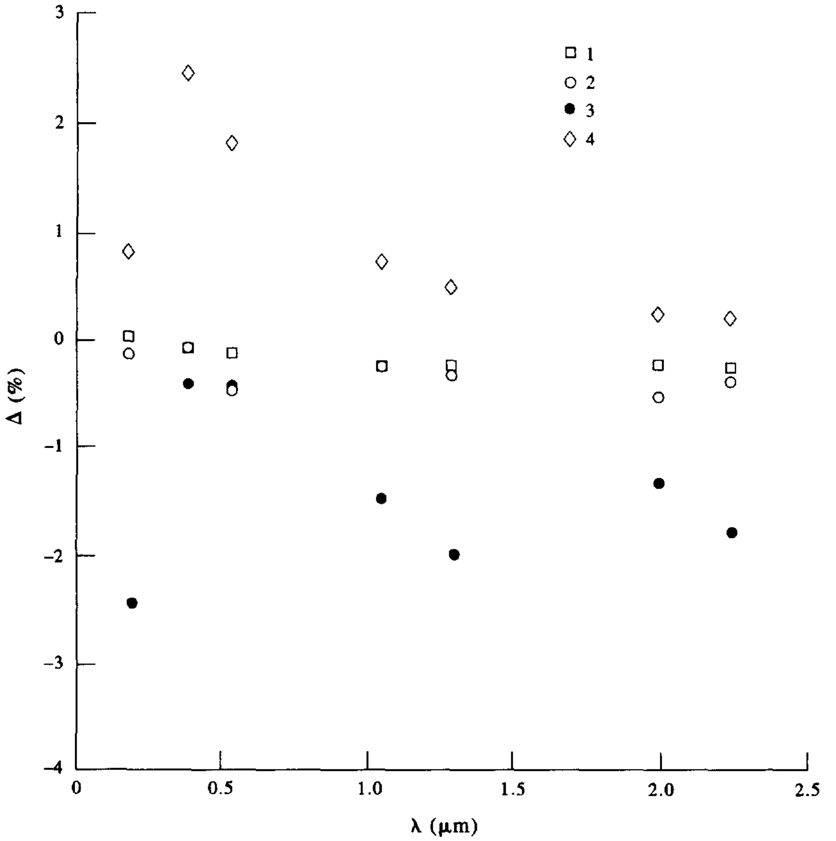

Fig. 2. The error of the value of σ_{ext} calculated using the Evans–Fournier approximation as a function of the wavelength λ for dust aerosol (1); oceanic aerosol (2); water-soluble aerosol (3) and soot aerosol (4).

Table 3. The parameters of the PSD (27) and a_{ef} for different aerosol media (Lenoble and Brogniez, 1984)

Aerosol	a^* (μm)	σ	a_{ef} (μm)
Dust	0.5	1.09527	10.0
Oceanic	0.3	0.92028	2.5
Water-soluble	0.005	1.09527	0.1
Soot	0.0118	0.69317	0.04

aerosols at wavelength $\lambda = 0.4$–$2.2\,\mu$m. The PSD assumed to be

$$f(a) = \frac{\exp\left(-\frac{1}{2\sigma^2}\ln^2\frac{a}{a^*}\right)}{\sqrt{2\pi}a\sigma}, \tag{27}$$

where the values of a^*, σ for different aerosols are presented in Table 3. One can see that the accuracy of the computation using the EFA is very high for all types of aerosol media. The relative error is less than 3%.

3. ABSORPTION EFFICIENCY FACTOR

The basic approximate solutions for the value of absorption efficiency factor Q_{abs} are summarized in Table 4. Their brief description is given below.

Table 4. The absorption efficiency factor Q_{abs} of a spherical particle ($c = 4\kappa x$, $M(c,b) = 2n^2c^{-2}(e^{-cb}(1+cb)-(1+c)e^{-c})$, $b = \sqrt{1-n^{-2}}$)

N	Approximation range of applicability	Q_{abs}
1.	Rayleigh, $x \ll 1$, $\lvert mx\rvert \ll 1$	$4x\,\mathrm{Im}\left[\dfrac{1-m^2}{2+m^2}\right]$
2.	Geometrical optics, $x \gg 1$, $\rho \gg 1$	
	Shifrin (1957)	$(1-S(n))(1-e^{-c})$
	Bohren and Nevitt (1983)	$\dfrac{4n(1-M(c,b))}{(n+1)-(n-1)^2e^{-c}}$
	Zege and Kokhanovsky (1988)	$F = 1 - M(c,b) - S(n)(1-e^{-cb})^2$
3.	Geometrical optics with regards to the edge effects (Kokhanovsky and Zege, 1994)	$F[1+(n-1)(1-e^{-1/t\rho})]$ for t see equation (35)
4.	Perelman, any x, $c \ll 1$, $\lvert m-1\rvert \ll 1$,	$\dfrac{16n^3}{c^2}\left(c\cosh\left(\dfrac{c}{2}\right)-\sinh\left(\dfrac{c}{2}\right)\right)$
5.	Rayleigh–Gans, $2x\lvert m-1\rvert \ll 1$, $\lvert m-1\rvert \ll 1$	$\dfrac{2c}{3}$
6.	van de Hulst, $x \gg 1$, $\lvert m-1\rvert \ll 1$	$1-2c^{-2}(1-(1+c)e^{-c})$

The value of Q_{abs} within the framework of the *Rayleigh approximation* coincides with the value of Q_{ext} (see Tables 1 and 4). It is associated with a strong absorption of radiation inside small absorbing particles as $x \ll 1$ and $\kappa \neq 0$. The value of the scattering efficiency factor Q_{sca} of such particles is very small.

The value of $Q_{abs} = 2c/3$ ($c = 4\kappa x$) within the *Rayleigh–Gans approximation* can be obtained from the Rayleigh formula as $m \to 1$. In this approach the value of the absorption coefficient $\sigma_{abs} = N\langle C_{abs}\rangle$ (N is the number concentration) does not depend on a particle radius but only on the volume of the absorbing matter (van de Hulst, 1957):

$$\sigma_{abs} = \alpha C_v, \tag{28}$$

where $\alpha = 4\pi x/\lambda$, $C_v = 4\pi N\int_0^\infty f(a)a^3\,da/3$ is the volumetric concentration. This important feature was discussed in more detail by many authors (van de Hulst, 1957; Shifrin, 1988; Lopatin and Sidko, 1988).

Note that equation (28) was extended to the case of layered spheres (Zege and Kokhanovsky, 1988) and ellipsoids (Kokhanovsky, 1991b).

In the *van de Hulst approximation* (van de Hulst, 1957; Ackerman and Stephans, 1987) the accuracy of the absorption efficiency factor Q_{abs}^{HA} (see Table 4) is less than that for Q_{ext}^{HA}. It rapidly decreases while the refractive index increases. A maximum of the error is found within the first maximum of the extinction curve $Q_{ext}(\rho)$. As $c = 4\kappa x \to 0$, van de Hulst formula passes in the Rayleigh–Gans solution (van de Hulst, 1957).

It was shown by Shifrin and Tonna (1992) that van de Hulst formula for Q_{abs} (see Table 4) can be approximated by the simple equation $Q_{abs} = 1 - \exp(-2c/3)$ with error less than 5%.

Note, these classical well-known approximations for the soft particles do not give solutions for any particle size.

Unlike these approaches, the *Perelman approximation* (see Table 4) provides the absorption efficiency factor Q_{abs} for any x, but for only week absorbing particles with $c \ll 1$ (see Table 4). As $c \to 0$ and $n \to 1$ Perelman's formula for Q_{abs} passes to the Rayleigh–Gans solution for Q_{abs} (Perelman, 1994).

Within the *geometrical optics approximation* the value of Q_{abs} is given by the following integral (Bohren and Huffman, 1983; Zege and Kokhanovsky, 1988):

$$I = \frac{1}{2}\sum_{j=1}^{2}\int_0^{\pi/2} E_j(\tau)\sin 2\tau\,d\tau, \tag{29}$$

where

$$E_j = \frac{(1-R_j)(1-e^{-c\delta})}{1-R_j e^{-c\delta}}, \qquad \delta = \sqrt{1-\frac{\cos^2\tau}{n^2}}, \tag{30}$$

$$R_1 = \frac{\sin^2(\tau-\tau')}{\sin^2(\tau+\tau')}, \qquad R_2 = \frac{\tan^2(\tau-\tau')}{\tan^2(\tau+\tau')}, \qquad c = 4\kappa x, \qquad \tau' = \arccos\left(\frac{\cos\tau}{n}\right). \tag{31}$$

There are a number of approximations for this integral. Some of them are given in Table 4.

We compared the results of calculations through different formulas in Fig. 3. The error of the Zege and Kokhanovsky's (1988) approximation (ZKA) is found to be approximately 1.5% at any c value. It is the best known approximation of integral I. As $n \to 1$, from the ZKA one can obtain the van de Hulst solution for Q_{abs}; as $c \to \infty$, the ZKA passes to the formula of the Shifrin's approximation (Shifrin, 1955), and as $c \to 0$, from the ZKA solution it follows the formula of Twomey and Bohren (1980) for weak absorbing spheres:

$$Q_{abs}^{GO} = 2c\psi(n)/3 \tag{32}$$

where $b = \sqrt{1-n^{-2}}$, $\psi(n) = n^2(1-b^3)$.

Shifrin and Tonna (1992) showed that the function $M(c, b)$ in Table 4 can be approximated by the following equation: $M(c, b) = \exp(-2c\psi(n)/3)$. Error of this approximation is less than 1% for $n \in [1.1, 1.5]$. They proposed to calculate the value of the absorption factor with the following solution: $Q_{abs} = 1 - \exp(-2\psi(n)c/3)$. The error of this approximation of integral I (see equation (29)) is less than 10% at $n < 1.5$ for any c. It is less than 5% at $c < 1$, $n < 1.5$.

Note, Shifrin's (1955) formula gives overestimation of the integral (29) (see Fig. 3). Being not very accurate in the geometrical optics limit ($x \gg 1$) it provides comparatively good accuracy for $n = 1.34$ and moderate particle sizes ($x = 20$–200). This explains good accuracy of this solution in the cloud optics (Feigelson, 1981; Khvorostjaninov and Khvorostjaninov, 1994; Mitchell and Arnott, 1994), where particles are not too large. To be more consistent, for expanding solutions over the moderate size particles, the edge terms should be accounted. Such approximate formula for the value of Q_{abs} was obtained by Nussenzveig

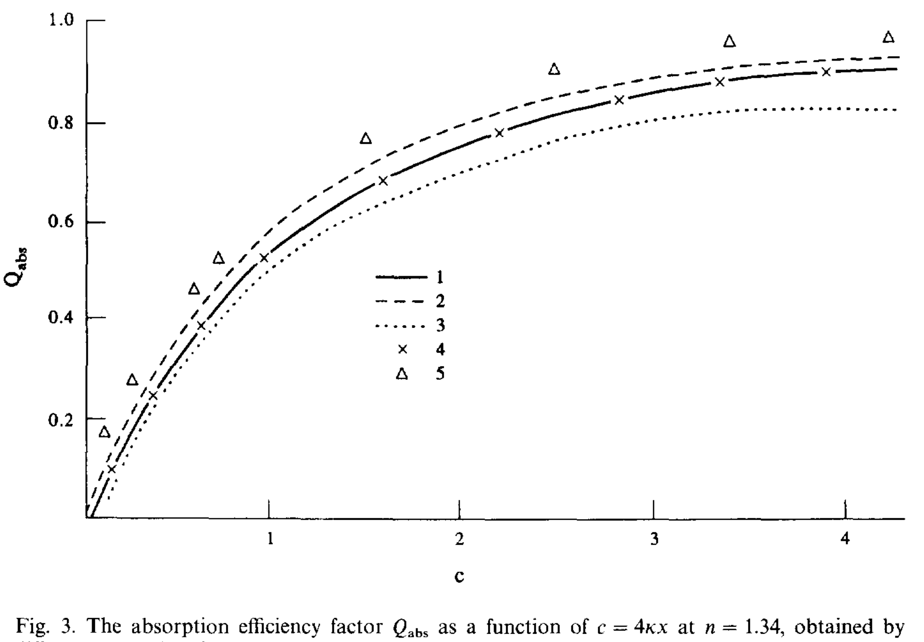

Fig. 3. The absorption efficiency factor Q_{abs} as a function of $c = 4\kappa x$ at $n = 1.34$, obtained by different approximations: 1, Zege and Kokhanovsky (1988); 2, Shifrin (1955); 3, Bohren and Nevitt (1983); 4, geometrical optics integral [see equation (29)]; 5, Mie calculation at $x = 20$–200, $\kappa = 10^{-3}$ ($c < 1$), $\kappa = 10^{-2}$ ($c > 1$).

and Wiscombe (1980), where the edge terms were given through some integrals. Unfortunately, the use of these very important results was hindered by the intricacy of the solution. Simple formulas for the edge terms were obtained by Kokhanovsky and Zege (1995) through the approximation of the Mie computation results. They were obtained at the value of $\kappa < 0.1$:

$$Q_{\mathrm{abs}} = T\left(1 - \frac{2n^2}{c^2}\left[\mathrm{e}^{-cb}(1+cb) - \mathrm{e}^{-c}(1+c)\right] - S(n)\left[1-\mathrm{e}^{-cb}\right]^2\right). \tag{33}$$

where

$$T = 1 + (n-1)(1 - \exp(-1/t\rho)), \tag{34}$$

$$t = [21.2 - 20.1\,z + 11.1\,z^2 - z^3]^{-4}, \qquad z = -lg\kappa, \qquad \rho = 2x(n-1). \tag{35}$$

Note the analytical solution for the function $S(n)$ (see (33)) was obtained by Gershun (1937) and independently by Acquista *et al.* (1980) (see Zolotova and Shifrin (1993) as well):

$$S(n) = \frac{8n^2(n^4+1)\ln n}{(n^4-1)^2(n^2+1)} - \frac{n^2(n^2-1)^2}{(n^2+1^3)}\ln\frac{n+1}{n-1}$$
$$\frac{3n^7 - 7n^6 - 13n^5 - 9n^4 + 7n^3 - 3n^2 - n - 1}{3(n^4-1)(n^2+1)(n+1)}. \tag{36}$$

The errors of Q_{abs} through equation (33) did not exceed 10% at $x \geqslant 10$ for typical aerosol refractive indices ($n = 1.2$–1.55), while the errors of formulae for Q_{abs} without account for edge affects can be as large as 30% (see Kokhanovsky and Zege, 1995).

Unlike the edge terms in equation (33), where coefficients were obtained from the Mie computation, the absorption cross section with edge terms for the weak absorbing ($c \to 0$) soft ($n < 1.2$) particles can be obtained directly within the complex angular momentum theory (Kokhanovsky, 1995):

$$Q_{\mathrm{abs}} = Q_{\mathrm{abs}}^{\mathrm{GO}} + Q_{\mathrm{abs}}^{\mathrm{E}}, \tag{37}$$

where the value of $Q_{\mathrm{abs}}^{\mathrm{GO}}$ is determined by equation (32) and the edge term

$$Q_{\mathrm{abs}}^{\mathrm{E}} = nc\left\{\arccos\left(\frac{1}{n}\right) - \frac{1}{n}\sqrt{1-\left(\frac{1}{n}\right)^2}\right\}. \tag{38}$$

From equations (7) and (37) it follows:

$$\sigma_{\mathrm{abs}} = A(n)C_v\alpha, \tag{39}$$

where $\alpha = 4\pi\kappa/\lambda$, C_v is the volumetric concentration,

$$A(n) = n^2(1-b^3) + \tfrac{3}{2}(n\arccos(n^{-1}) - b). \tag{40}$$

As $n \to 1$, $A(n) \to 1$ and from equation (39) one can obtain equation (28).

The error of equation (39) is less than 6% at $n < 1.2$, $\kappa < 10^{-4}$ at $\lambda = 0.55\,\mu$m and $\alpha_{\mathrm{ef}} = 1$–$10\,\mu$m (see Fig. 4). Equation (37) provides more accurate data than the van de Hulst and Perelman approximations at larger n.

4. ASYMMETRY PARAMETER

The asymmetry parameter (average cosine of the phase function g) is determined by equation (5). This value is used in many solutions of the radiative transfer equation (Sobolev, 1975; van de Hulst, 1980; Ishimaru, 1981; Zege *et al.*, 1990, Zege and Katsev, 1991). Multiple scattering process mainly depends on this only integral characteristics of the phase function. Moreover, there are correlations between the asymmetry parameter and other optical characteristics of the phase function $p(\theta)$, used in various solutions of the

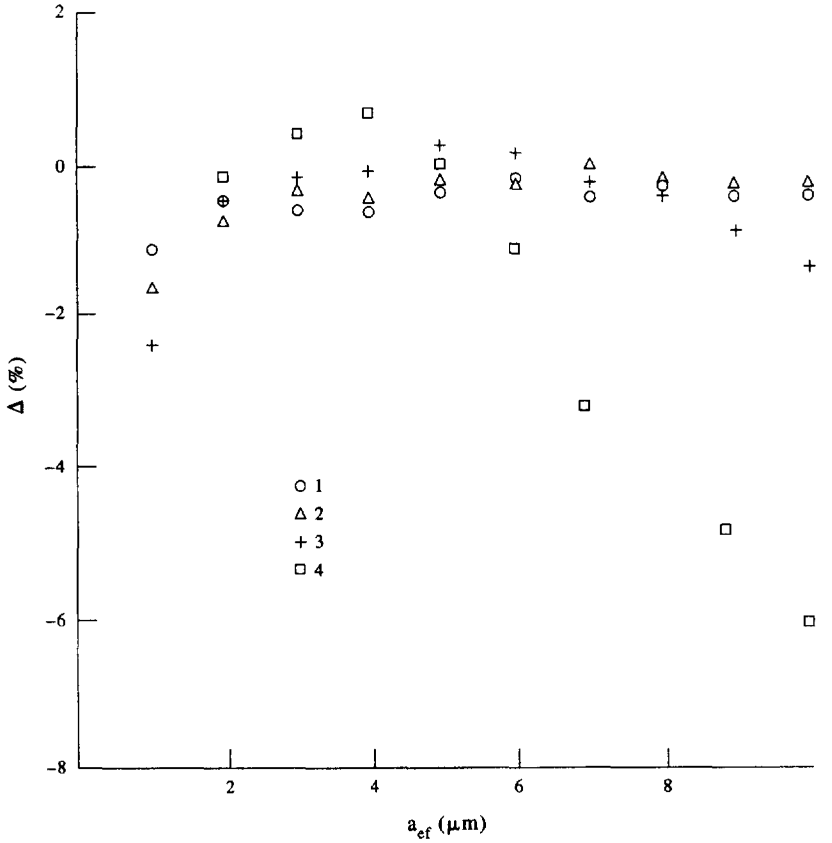

Fig. 4. The error Δ of the value of σ_{abs} calculated using equation (39) as a function of the effective radius a_{ef} at $\kappa = 10^{-5}$, $\lambda = 0.55\,\mu$m and $n = 1.05$ (1), 1.1 (2), 1.15 (3),1.2 (4).

radiative transfer theory. As example, the following simple correlations between the portion of light scattered into the backward semisphere F and the mean squared scattering angle β_2 with the value of g were obtained (Zege *et al.*, 1990):

$$F \approx \frac{1-g}{3}, \qquad \beta_2 \approx 1 - g \tag{41}$$

for typical ocean and cloud phase functions. Here

$$F = \frac{1}{2}\int_{\pi/2}^{\pi} p(\theta)\sin\theta\,d\theta, \qquad \beta_2 = \frac{\int_0^{\pi/2} \theta^3 p(\theta)\,d\theta}{\int_0^{\pi/2} \theta p(\theta)\,d\theta} \tag{42}$$

Basic approximate formulas for the asymmetry parameter g are presented in Table 5. Let us comment them.

It is evident that for the *Rayleigh scattering* the asymmetry parameter $g = 0$.

Within the *Rayleigh–Gans approximation* the asymmetry parameter does not depend on the refractive index (Irvine, 1963). It is given as a ratio of $\varphi(x)$ to $H(x)$ in Table 4. The function $\varphi(x)$ has already been introduced above by equations (19)–(21) and

$$H(x) = \frac{4}{x^4}\left\{\left(\frac{9}{128} - \frac{5x^2}{64}\right)(4x\sin 4x + \cos 4x) + \frac{x^6}{2} + \frac{11x^4}{8} - \frac{31x^2}{64} - \frac{9}{128} + x^2\left(x^2 - \frac{3}{8}\right)(Ci(4x) - \gamma - \ln(4x))\right\}. \tag{43}$$

The van de Hulst and Perelman approximations do not provide analytical solutions for the asymmetry parameter g. In the first case, to obtain the value of g one has to calculate two integrals (van de Hulst, 1957).

Table 5. The asymmetry parameter g for a spherical particle

N	Approximation, range of applicability	g
1.	Rayleigh, $x \ll 1$, $\|mx\| \ll 1$	0
2.	Geometrical optics, $x \gg 1$, $\rho \gg 1$	
	van de Hulst (1957)	$\frac{1+v\xi}{1+\xi}$ for ξ and v see equations (45) and (46)
	Zege and Kokhanovsky (1988)	$g_\infty - (g_\infty - g_0)e^{-cy}$
3.	Geometrical optics with regards to the edge effects (Kokhanovsky and Zege, 1995)	$g_\infty - \left(g_\infty - g_0 + \frac{\gamma_1 + \gamma_2 c}{x^{2/3}}\right)e^{-cy}$ y, g_∞, g_0 are given in Table 6.
4.	Rayleigh–Gans	$H(x)/\varphi(x)$ for $\varphi(x)$ and $H(x)$ see equations (19)–(21) and equation (43)
	$m - 1 \ll 1$ $x\|m-1\| \ll 1$	$x \to 0$: $0.16x^2$; $x \to \infty$: $1 - x^{-2}\ln 4x$

Within the *geometrical optics* approximation it follows (van de Hulst, 1957; Irvine, 1965; Zege and Kokhanovsky, 1988):

$$g = \frac{1 + v\xi}{1 + \xi}, \tag{44}$$

where

$$v\xi = \frac{1}{2}\sum_{j=1}^{2}\int_0^{\pi/2} \frac{d_j(\tau)\sin 2\tau\, d\tau}{1 - 2R_j e^{-c\delta}\cos 2\tau' + R_j e^{-2c\delta}}, \tag{45}$$

$$\xi = \frac{1}{2}\sum_{j=1}^{2}\int_0^{\pi/2} \left\{R_j + \frac{(1-R_j)^2 e^{-c\delta}}{1 - R_j e^{-c\delta}}\right\}\sin 2\tau\, d\tau, \tag{46}$$

$$d_j(\tau) = e^{-c\delta}(1 - R_j)^2 \cos 2(\tau - \tau') + R_j \cos 2\tau(1 - e^{-2c\delta})$$
$$+ 2R^2(e^{-c\delta} - \cos 2\tau')\cos 2\tau e^{-c\delta},$$

$$\cos\tau' = \cos\tau/n, \qquad \delta = \sqrt{1 - \cos^2\tau/n^2}. \tag{47}$$

Note, in this approximation, the scattering efficiency factor Q_{sca} is

$$Q_{\text{sca}} = 1 + \xi. \tag{48}$$

Zege and Kokhanovsky (1988) suggested the approximation of equation (44) by the following formula

$$g = g_\infty - (g_\infty - g_0)e^{-cy}, \tag{49}$$

where g_∞ is the asymmetry parameter of a strong absorption particle ($c \to \infty$), and $g_0(n)$ is the asymmetry parameter of a transparent particle ($c = 0$). The functions $g_0(n)$, $g_\infty(n)$, $y(n)$ along with the function $S(n)$, that appeared in equation (36) are presented in Table 6.

The edge terms for light pressure efficiency Q_{pr} were obtained by Nussezveig and Wiscombe (1980), but unfortunately, in too complicated form. Relied upon these results, we suggested the following approximation for the asymmetry parameter (Zege and Kokhanovsky, 1988):

$$g = g_\infty - \left(g_\infty - g_0 + \frac{\gamma_1 + \gamma_2 c}{x^{2/3}} e^{-cy}\right). \tag{50}$$

Table 6. Parameters S, g_0, g_∞ and y at different values of n (Zege and Kokhanovsky, 1988)

n	S	g_0	g_∞	y
1.1	0.0252	0.9731	0.9946	0.5180
1.2	0.0443	0.9341	0.9856	0.6528
1.25	0.0529	0.9147	0.9806	0.6948
1.3	0.0611	0.8961	0.9751	0.7280
1.333	0.0664	0.8843	0.9714	0.7468
1.34	0.0675	0.8818	0.9706	0.7505
1.35	0.0691	0.8783	0.9695	0.7555
1.4	0.0768	0.8613	0.9638	0.7785
1.45	0.0844	0.8542	0.9579	0.7985
1.5	0.0918	0.8299	0.9520	0.8160
1.55	0.0991	0.8154	0.9460	0.8315
1.60	0.1063	0.8015	0.9400	0.8453
1.65	0.1133	0.7884	0.9340	0.8580
1.70	0.1203	0.7759	0.9280	0.8695
1.90	0.1475	0.7314	0.9046	0.9080
2.00	0.1606	0.7121	0.8933	0.9340
2.10	0.1734	0.6945	0.8823	0.9383

The values of coefficients γ_j were found with Mie computation. It appeared that $\gamma_1 = 0.5$, $\gamma_2 = 0.2$ for aerosol particle at $1.2 \leqslant n \leqslant 2.0$. The analytical solution for the value of g_∞ was obtained by Barun and Gavrilovich (1987) at $\kappa \ll n$:

$$g_\infty = \frac{1 + g_1 \ln n - g_2 \ln((n+1)/(n-1)) + g_3}{1 + S}, \tag{51}$$

where

$$g_1 = \frac{8n^4(n^6 - 3n^4 + n^2 - 1)}{(n^4 - 1)^2(n^2 + 1)^2}, \tag{52}$$

$$g_2 = \frac{(n^2 - 1)^2(n^8 + 12n^6 + 54n^4 - 4n^2 + 1)}{4(n^2 + 1)^4}, \tag{53}$$

$$g_3 = \frac{\sum_{j=1}^{12} B_j n^j}{24(n^2 + 1)^2(n^4 - 1)(n + 1)}, \tag{54}$$

$$B_j = (-3,\ 13,\ -89,\ 151,\ 186,\ 138,\ -282,\ 22,\ 25,\ 25,\ 3,\ 3). \tag{55}$$

The errors of equation (50) as well as equations (14) and (33) were studied by Zege and Kokhanovsky (1992). Some results of this investigation are presented in Figs 5 and 6 and in Table 7.

One can see that the errors of these approximate formulae is are less than 10–15% for coarse aerosols ($x > 10$–60).

Note, it is not occasionally that we study the accuracy of the value of $1 - g$ but not g. Just this value appears in the radiative transfer equation solution and defines its accuracy. The relative errors of calculation of the value of asymmetry parameter g is less than 1–2%.

5. SPHERICAL POLYDISPERSIONS, WATER CLOUDS

Real scattering media usually consist of scatterers of different sizes, characterized by the particle size distribution. To obtain optical parameters of such media using the above results one needs:

—to specify the particle size distribution of an aerosol and the complex refraction index of particles in the studied spectral range;

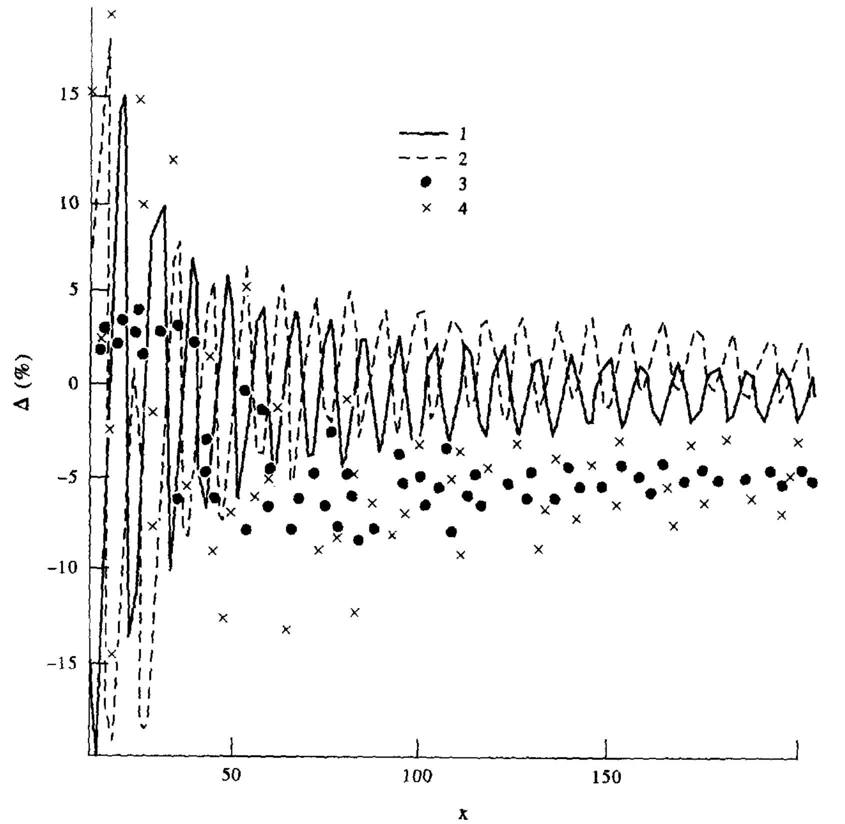

Fig. 5. Errors of the value of Q_{ext} (1), $1-g$ (2), Q_{abs} (3), $1-\omega$ (4) calculated by equations (14), (33) and (50) at $n = 1.34$ and $\kappa = 10^{-4}$ as a function of the size parameter x.

Table 7. The boundary x^* of a range $x > x^*$, where relative errors of values of Q_{ext}, $1-g$, Q_{abs}, through formulae (14), (50) and (33), respectively, are less than 10% (Zege and Kokhanovsky, 1992)

n	1.2			1.34			1.55		
κ	10^{-4}	10^{-3}	10^{-2}	10^{-4}	10^{-3}	10^{-2}	10^{-4}	10^{-3}	10^{-2}
Q_{ext}	35	35	25	45	45	20	10	10	10
$1-g$	60	50	40	35	35	35	25	25	12
Q_{abs}	10	10	10	10	10	10	10	10	10

—to choose the formulae from Tables 1, 4 and 5 for the specific particle sizes and refractive index;

—to integrate the chosen formulae with the particle size distribution in accordance with equations (7)–(9).

This recipe is simple and reliable. Nevertheless, sometimes the results being much simpler than Mie computations are not so simple as one would like. But unlike the Mie theory, this approach permits the future simplifications.

Below we are going to give the only example, where this way brings to the simple and accurate formulae for cloud optical properties. Just this example is interesting because of the great importance of the problem for climate study and cloud remote sensing.

What are water clouds from the point of view of the scattering theory?

First of all they are the polydispersions of the spherical particles. Effective radii of water drops appear to range from 4 to 20 μm (Mazin, 1989). So, in the visible wavelength region clouds can be considered as particulate media with large scatterers ($a \gg \lambda$). Meanwhile, in the near infrared region of the spectrum, the edge effects should be accounted for. So, to calculate the values of $\sigma_{ext} = N\langle C_{ext}\rangle$, $\sigma_{abs} = N\langle C_{ext}\rangle$ and $\langle\cos\theta\rangle = \langle gC_{sca}\rangle/\langle C_{sca}\rangle$ of

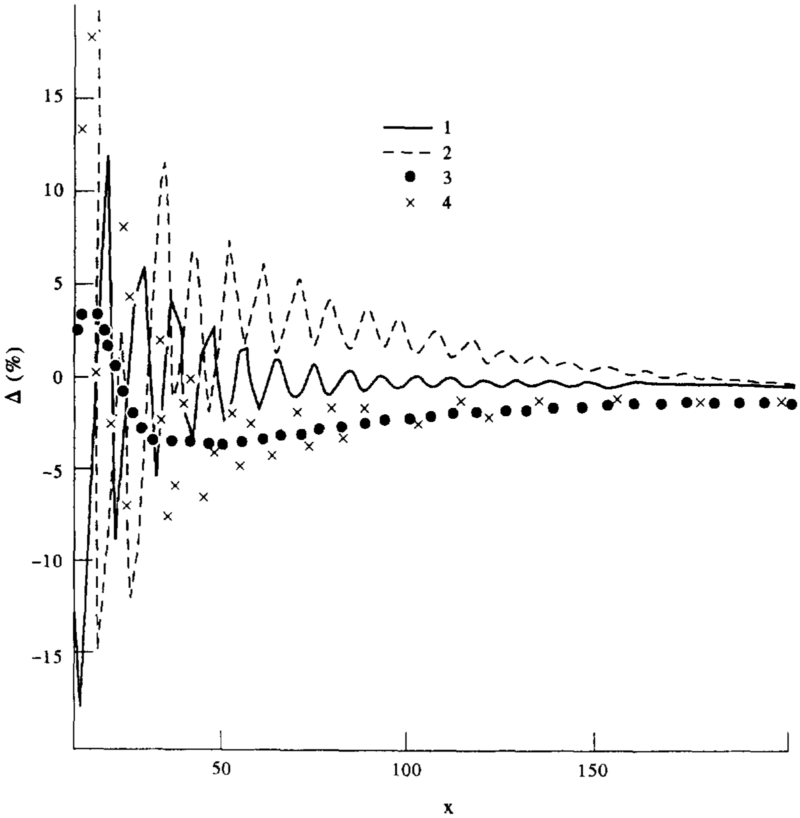

Fig. 6. Errors of the value of Q_{ext} (1), $1-g$ (2), Q_{abs} (3), $1-\omega$ (4) calculated by equations (14), (33) and (50) at $n = 1.34$, $\kappa = 10^{-2}$ as a function of size parameter x.

water clouds in the visible and near-infrared regions of the spectrum, the GO formulae with edge terms (14), (33) and (50) are suitable. Note, these solutions imply any absorption of light inside a particle. The solutions for the extinction and absorption coefficients as well as for the asymmetry parameters for such polydispersions with gamma particle size distribution (Mazin, 1989), were obtained by Kokhanovsky and Zege (1995). The gamma function is the usual model of the PSD in the cloud optics (Mazin, 1989). Regarding the specific features of the water clouds allows the additional simplification. First of all, water clouds are weak absorbing. Besides, the real part of the index of refraction of water in the near infrared region of the spectrum changes slowly ($n = 1.29$–1.34) (Halle and Querry, 1973). It results in small changes of the functions $y(n)$, $g_\infty(n)$, and $g_0(n)$ within this range these functions can be replaced by constants: we have used the values of $y(1.34)$, $g_0(1.34)$, $g_\infty(1.34)$ (see Table 5) for all λ in the near IR and visible bands. With this in mind, it was found (see Kokhanovsky and Zege, 1995):

$$\sigma_{ext} = \frac{1.5\,C_v}{a_{ef}}\left(1 + \frac{1}{(ka_{ef})^{2/3}}\right), \tag{56}$$

$$\sigma_{abs} = \frac{5\pi\kappa}{\lambda} C_v(1 - \alpha a_{ef})\left[1 + 0.34\left(1 - \exp\left(-\frac{8\lambda}{a_{ef}}\right)\right)\right], \tag{57}$$

$$1 - \langle\cos\theta\rangle = 0.12 + 0.5x_{ef}^{-2/3} - 0.15\alpha a_{ef}. \tag{58}$$

Simple formulae (56)–(58) give the local optical characteristics of water clouds in the visible and near infrared bands (Kokhanovsky and Zege, 1995).

As it follows from equations (56)–(58) all considered optical parameters of clouds depend on the only PSD parameter a_{ef}. This interesting feature of the cloud optics has been mentioned more than once (Hansen and Travis, 1974; Damiano and Chylek, 1994), but

the above solutions display it clearly. A comparison between the data obtained through equations (56)–(58) and the Mie theory was performed for water droplets at $\lambda = 0.45$–$2.25\,\mu$m for the gamma PSD (Deirmendjian, 1969) at $a_{ef} = 4 - 2.20\,\mu$m, $\mu = 6$ and for the lognormal PSD (27) at $a_{ef} = 6\,\mu$m ($a^*_{ef} = 4.49\,\mu$m, $\sigma = 0.34$). The relative errors of approximations for σ_{ext}, σ_{abs}, $1 - \omega_0$, $1 - \langle\cos\theta\rangle$ values in all cases are less than 5–10% (see Figs 7 and 8). Moreover, from these data it follows that equations (56)–(58) could be used not only for gamma PSD, but for the lognormal PSD also. It is interesting that formulae (56)–(58) can be obtained from equations (14), (33) and (50) replacing a by a_{ef} (as $\alpha a_{ef} \to 0$).

6. CONCLUSIONS

We have presented here the system of the analytical solutions for the efficiency factors of extinction, absorption, and the asymmetry parameter of the phase function at different values of the index of refraction and size parameter. These formulas are accompanied with the accuracy estimation and the range of applicability.

Classical approximations (Rayleigh, Rayleigh–Gans, van de Hulst) are presented along with new approaches, never mentioned before in the reviews and books. Among them are the Perelman approximation and the geometrical optics solution with account for the edge terms, developed by the authors recently.

You do not need to be an expert in the scattering theory to read and use our stuff. Our potential customers are researchers and engineers who deal with different fields of aerosol optics, optical particle sizing, climate problem as well as remote sensing of aerosols and

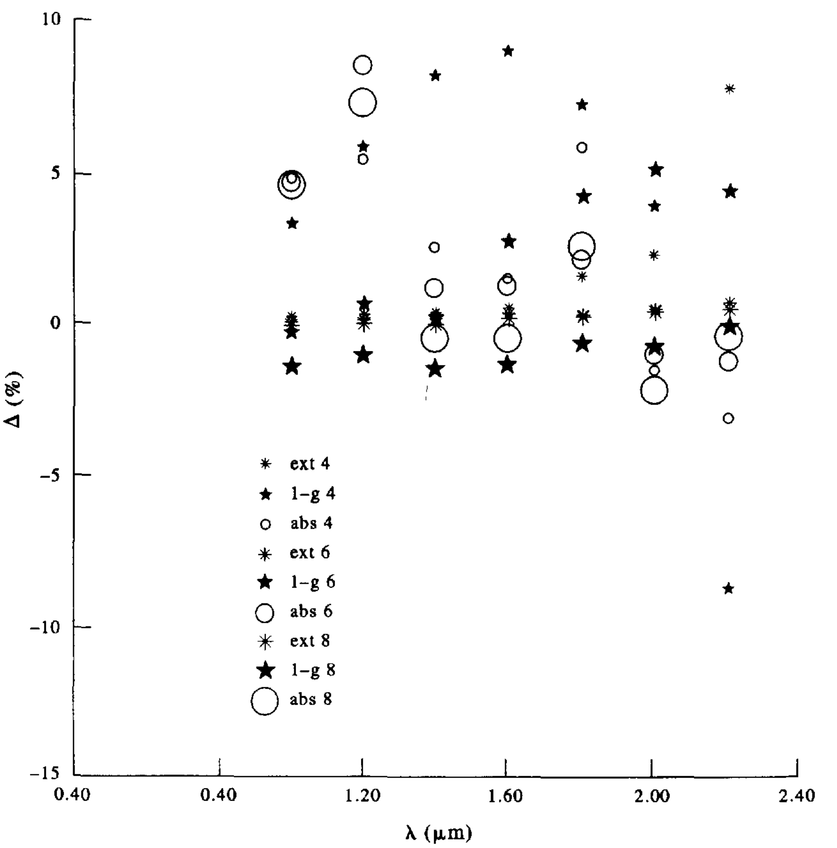

Fig. 7. The error of calculation of the values of σ_{ext}, $1 - \langle\cos\theta\rangle$ and σ_{abs} by equations (56)–(58) as a function of the wavelength λ at $\mu = 6$, $a_{ef} = 4, 6, 8\,\mu$m.

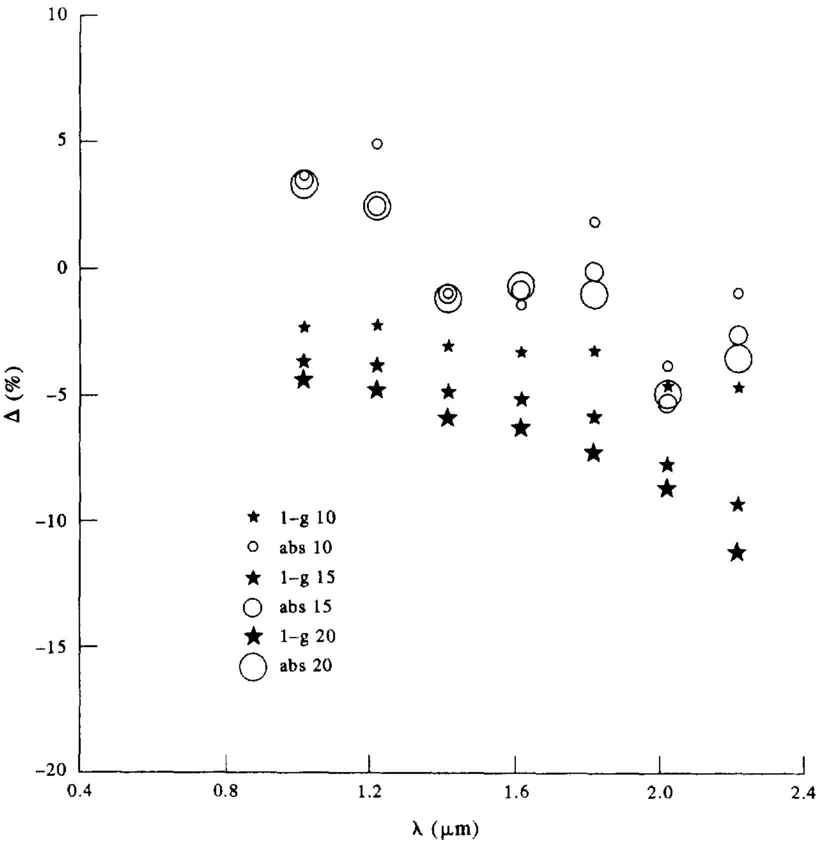

Fig. 8. The error of calculation of the values of σ_{ext}, $1 - \langle \cos\theta \rangle$ and σ_{abs} by equations (56)–(58) as a function of wavelength λ at $\mu = 6$ and $a_{ef} = 10, 15, 20\,\mu$m.

clouds. All of them need simple straightforward relations between the optical and microphysical parameters instead of (or along with) the Mie computation algorithms.

We believe this review is the most up-to-date and user-friendly and will be useful for aerosol community. In particular, we are sure, that it will be instructive for the researches to employ equations (56)–(58) in the cloud–radiation interaction studies.

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